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Workshop on Nonlinear Water Waves and Related Topics

http://murasige.sci.ibaraki.ac.jp/WS_Nonlinear_Water_Waves_Shonan_Village_Center_2023.html

Date : November 6 (Mon) – 10 (Fri), 2023

Place : Shonan Village Center (Hayama, Kanagawa, Japan)

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Program with title & abstract

November 6 (Mon)

14:00-14:10 *Opening*

14:10-14:50 Hidetaka Houtani (University of Tokyo)

Pressure field evaluation beneath nonlinear waves using the higher-order spectral method and its application to ship response simulations

The higher-order spectral method (HOSM) has been widely used to study the nonlinear evolution of water waves, especially in the context of the probability of freak wave occurrence. The HOSM efficiently solves the temporal evolution of the free-surface elevation and the velocity potential on the free surface under spatially periodic boundary conditions. However, to simulate ship responses in such nonlinear waves, it is necessary to evaluate the flow fields, i.e., the pressure fields, beneath the free surface. This study evaluated the flow fields beneath the free surface for unidirectional irregular waves using HOSM outputs. We discuss the validity of the evaluated flow fields in terms of the nonlinearity order and the degree of satisfaction of the free surface boundary conditions of the HOSM simulation. In addition, we present some findings on the ship response statistics in such nonlinear irregular waves simulated using the HOSM-derived flow fields beneath the free surface.

14:50-15:30 Ricardo Barros (Loughborough University)
Mode-2 internal solitary waves in stratified fluids

Internal waves are commonly observed across the world ' s oceans and are of great importance in physical oceanography. Despite approximately 90% of the kinetic energy associated with oceanic internal waves residing in the first two modes, mode-2 waves have received limited attention. This can be attributed, in part, to the greater mathematical complexity associated with higher modes. In this talk we will discuss mode-2 localised internal solitary waves in a three-layer configuration, bounded above and below by a rigid wall. We will start by considering a strongly nonlinear long wave model that extends the two-layer Miyata-Choi-Camassa (MCC) model. Its solitary-wave solutions are governed by a Hamiltonian system with two degrees of freedom, and revealed by Barros et al. (2020) to have several strongly nonlinear characteristics that fail to be captured by the existing weakly nonlinear theory, including new solutions characterised by multi-humped profiles. Such solutions are compared with those obtained for the fully nonlinear Euler equations. We will then discuss ongoing work on more realistic configurations where the density of the fluid varies continuously with depth.

References :

R.Barros, W.Choi, P.A.Milewski (2020) Strongly nonlinear effects on internal solitary waves in three-layer flows. J. Fluid Mech. 883 A16.

A.Doak, R.Barros, P.A.Milewski (2022) Large mode-2 internal solitary waves in three-layer flows JFM, J. Fluid Mech. 953 A42.

Coffee break

15:50-16:30 Nicholas Pizzo
(Scripps Institution of Oceanography, UC San Diego)
The role of Lagrangian drift in the geometry, kinematics and dynamics of surface waves

The role of the Lagrangian mean flow, or drift, in modulating the geometry, kinematics and dynamics of rotational and irrotational deep-water surface gravity waves is examined. A general theory for permanent progressive waves on an arbitrary vertically sheared steady Lagrangian mean flow is derived in the Lagrangian reference frame and mapped to the Eulerian frame. A Lagrangian viewpoint offers tremendous flexibility due to the particle labelling freedom and allows us to reveal how key physical wave behaviour arises from a kinematic constraint on the vorticity of the fluid, inter alia the nonlinear correction to the phase speed of irrotational finite amplitude waves, the free surface geometry and velocity in the Eulerian frame, and the connection between the Lagrangian drift and the Benjamin– Feir instability. To complement and illustrate our theory, a small laboratory experiment demonstrates how a specially tailored sheared mean flow can almost completely attenuate the Benjamin–Feir instability, in qualitative agreement with the theory. The application of these results to problems in remote sensing and ocean wave modelling is discussed. We provide an answer to a long-standing question: remote sensing techniques based on observing current-induced shifts in the wave dispersion will measure the Lagrangian, not the Eulerian, mean current.

16:30-17:10 Wooyoung Choi (New Jersey Institute of Technology)
On resonant triad interactions between surface and internal waves

We consider resonant triad interactions between surface and internal gravity waves in a two-layer system with a free surface. As the system supports both surface and internal wave modes, two different types of resonant triad interactions are possible: one with two surface and one internal wave modes and the other with one surface and two internal wave modes. For both types of resonances, an explicit Hamiltonian system is studied numerically and its solutions are compared with the numerical solutions of the reduced amplitude equations. Some possible oceanic applications are discussed.

November 7 (Tue)

9:00-9:40 Christopher Curtis (San Diego State University)
Modeling and Analysing Nonlinear Wave Fields

In this talk, we focus on developing models of and analysing more complex flows in which isolated coherent structures are either nonexistent or difficult to detect due to high variations in the background flow. In the first part of the talk, we present a recently derived modification of the Dysthe equation which takes into account constant vorticity. While still a relatively simple model of the physics, we show how higher-order effects and vorticity can substantially alter the statistical properties of the flow in comparison to the more standard nonlinear Schroedinger equation. In particular, we present numerical studies of the kurtosis of wavefield distributions as a function of the Benjamin–Feir index and background vorticity, thereby showing how stability metrics and constant sheer influence the potential for rare events in deep water flows. In the second part of this talk, moving away from developing models from first principles, we look at using the measurement based dynamic-mode decomposition to detect and characterize localized vortices in weakly turbulent background flows in the 2D defocusing nonlinear Schroedinger equation. With this, we are able to numerically demonstrate the formation of relatively stable subspaces of modes which characterize most of the flow, hinting at the potential of relatively low dimensional structures persisting in otherwise chaotic conditions. Finally, moving from modal analysis to data-driven modeling, we show how the dynamic-mode decomposition coupled with techniques from machine learning can be used to construct models which accurately recreate and predict dynamics in the Kuramoto–Sivashinksy equation. Thus, we see how data-driven techniques can be used to both analyse and model complex flow conditions without making recourse to physical modeling.

9:40-10:20 Naoto Yokoyama (Tokyo Denki University)
Anisotropic energy cascade in strongly stratified turbulence

In broad-band anisotropic turbulence such as stratified turbulence, anisotropic weak-wave turbulence and isotropic Kolmogorov turbulence as well as large-scale flows often coexist. We have proposed a procedure to uniquely determine the anisotropic energy-flux vectors in the wavenumber space based on two ansatzes, the net locality of the energy balance and the efficiency of the energy transfer. In this talk, the procedure is applied to strongly stratified turbulence, and the local anisotropic energy-flux vectors numerically obtained from a direct numerical simulation are compared with the well-known nonlocal interactions referred to as parametric subharmonic instability, elastic scattering, and induced diffusion.

Coffee break

10:40-11:20 Mitsuhiro Tanaka (Gifu University)
Numerical Study of Breaking Phenomena in the Fornberg-Whitham Equation

The Fornberg-Whitham equation (FW eq.) is well-known as a simple model equation capable of reproducing the phenomena of peaking and breaking both of which are characteristic phenomena of nonlinear surface gravity waves.
In this study, we investigate numerically the shape of the critical curve in the space of the parameters involved in the FW eq. which delineates the region where breaking occurs during the time evolution of the solutions versus the region where it does not.
Furthermore, we aim to qualitatively understand the shape of the critical curve from the perspective of the competition between nonlinearity and dispersion.

11:20-12:00 Takeshi Kataoka (Kobe University)
Exponential asymptotics and related wave phenomena

An exponential asymptotic analysis has recently been used to analyze wave phenomena whose amplitude is exponentially small. For example, ship waves or surface waves due to submerged sources at small Froude numbers ($F \ll 1$) have amplitudes that are exponentially small with respect to F. The same is true for inertia–gravity waves induced in the atmosphere or ocean by balanced geostrophic flow at low Rossby numbers ($R \ll 1$). In both cases, the assumption of linearization is not appropriate and the role of nonlinear effects at small F or R depends on exponential (‘ beyond-all-orders ’) asymptotics. However, the existing exponential asymptotic technique is only applicable to ordinary differential equations. We have developed a new exponential asymptotic technique that is applicable to partial differential equations, which is discussed here in the context of a simple partial differential equation with a quadratic nonlinear term. (This is a joint work with T. R. Akylas (MIT))

Lunch

14:00-14:40 Zhan Wang (Chinese Academy of Sciences)

Interfacial waves under horizontal electric fields: Hamiltonian structure and bifurcation of solitary waves

Electrohydrodynamics, an interdisciplinary subject coupling electrostatic fields into fluid flows through the Maxwell stress tensor, has a wide range of applications in chemistry and industry, including the cooling system, coating process, and microfluidic device. The tangential electric field, parallel to the undisturbed interface, has a stabilizing effect since it provides a dispersive/dissipative contribution to the linear system. It can delay the formation of the film rupture and even suppress the Rayleigh-Taylor instability. This talk focuses on interfacial waves between two dielectrics under tangential electric fields in the inviscid limit, which can be formulated as a Hamiltonian system. Asymptotic modelling based on the Hamiltonian framework, as well as the direct numerical simulations of the primitive equations, are combined to capture the nonlinear features of the system. The tangential electric field leads to a more complicated dispersion relation, and its stabilizing nature expands the range of parameters that can be explored; hence, peculiar wave phenomena, particularly new types of solitary waves, are discovered.

14:40-15:20 Paul Milewski (The Pennsylvania State University)

Resonant free-surface water waves in closed basins

Nonlinear resonance is a mechanism by which energy is continuously exchanged between a small number of linear wave modes, a phenomenon common to a large number of dispersive fluid systems. In the context of free-surface gravity waves, nonlinear resonance has been studied extensively over the past century, with a particular focus on domains that are large compared to the characteristic wavelength (such as oceans) where only quartic resonances occur. However, nonlinear resonance in confined three-dimensional geometries has received relatively little attention. We will present the results of a combined theoretical and computational investigation into the characterisation and dynamics of resonant triads in cylindrical basins. Extensions and applications will be discussed.

Coffee break

15:40-16:20 Roberto Camassa (University of North Carolina)

Fluid-boundary interaction: confinement effects, stratification and transport

Arguably some of the most interesting phenomena in fluid dynamics, both from a mathematical and a physical perspective, stem from the interplay between a fluid and its boundaries. This talk will present some examples of how boundary effects lead to remarkable outcomes. Singularities can form in finite time as a consequence of the continuum assumption when material surfaces are in smooth contact with horizontal boundaries of a fluid under gravity. For fluids with chemical solutes, the presence of boundaries impermeable to diffusion adds further dynamics which can give rise to self-induced flows and the formation of coherent structures out of scattered assemblies of immersed bodies. These effects can be analytically and numerically predicted by simple mathematical models and observed in " simple " experimental setups.

16:20-17:00 John Grue (University of Oslo)

Upstream influence

Wave effects from ships moving along shallow waterways continue to be of interest. Ship speed relative to shallow water speed expressed by the Froude number is typically subcritical but may in certain circumstances become critical and is investigated here. At critical speed, water continuously piles up at the ship bow. The wave energy spreads laterally. If the water is horizontally unbounded, a frontal system of three-dimensional, curved solitary waves is developed and has been computed using nonlinear long wave theories (KP-equation, Boussinesq equations, other). Likewise, subsea slides may move at critical speed. Similar upstream wave system is generated as in the ship case, but is less investigated. Questions that concern the wave generation phase at critical speed condition are addressed: the role of dispersion; distance between the upstream waves, geometry characteristics when the generation happens. Strongly nonlinear and fully dispersive method in three dimensions is used to model the ship and subsea slide generation. Two scenarios are studied in each of the cases, where elevation due to high pressure (ship/slide) is compared to the corresponding low pressure case. In the latter condition, the upstream leading elevation is negative and moves at speed less than 1. The nonlinear-dispersive waves are found at the location of the generating source. The incipient wavelength in the high/low pressure cases are investigated and found to be the same, or almost the same. This illustrates the important role of dispersion in upstream generation at critical speed.

November 8 (Wed)

9:00-9:40 Taro Kakinuma (Kagoshima University)
A numerical study on water wave generation due to air pressure waves

Tsunamis were widely observed when the large eruption of Hunga Tonga–Hunga Ha ʻ apai volcano occurred in January 2022. In the present study, numerical simulations were generated using a nonlinear shallow-water model of velocity potential to study the fundamental processes of tsunami generation and amplification by air pressure waves over seabed topography. Both surface and internal waves generated by an air pressure wave were also simulated when the traveling speed of the air pressure wave is close to the phase velocity of surface or internal wave mode in shallow water.

9:40-10:20 Amin Chabchoub (Kyoto University)
Modulation Instability in Standing Water Waves

The modulation instability (MI) is responsible for the formation of strong wave localizations on the water surface and in a variety of nonlinear dispersive wave systems. Recent laboratory studies confirmed the validity of breathers to describe several MI scenarios in hydrodynamics, optics, plasma, and Bose-Einstein condensates. We provide experimental evidence of nonlinear and distinct breather focusing in standing water waves, ie, in two counterpropagating wave trains. The wave measurements are in excellent agreement with the hydrodynamic coupled nonlinear Schrödinger equation and indicate that MI processes are unimpeded during the interplay of two wave systems.

Coffee break

10:40-11:20 Takuji Waseda (University of Tokyo)

Nonlinear evolution of waves propagating under sea ice

Ocean waves are characterized by a narrow directional spectrum, and in the case of growing conditions under wind, the spectral peak downshifts. In this presentation, we show observational and experimental evidence of a fast spectral downshift of the wave spectrum in a decaying wave field. In the Sea of Okhotsk in 2020, we deployed a wave buoy on ice that measured attenuated swell propagating through a Marginal Ice Zone. The incoming wave field was measured by a stereo camera system on board P/V Soya. The observation showed that the spectral peak period downshifted from about 10 s to 15-17 s propagating through an MIZ for about 60 km as the wave energy reduced by 1/2. We conjectured that this anomalous spectral downshift is caused by dissipation-driven nonlinear energy transfer. Partly inspired by this observation, we have conducted a wave tank experiment at the U. of Tokyo. The tank is 8m long, 1.5 m wide, and 0.6 m depth, equipped with a plunging wave breaker. The tank is filled with fresh water. By generating waves while freezing the water at room temperature in the range of -7.5 to -5 ℃, “frazil ice” forms. A modulated wave train was generated and its propagation under the “frazil ice” field was investigated. The modulated wave train was generated by seeding a sideband perturbation to a monochromatic wave train. The wave train rapidly loses energy while shifting energy to the lower frequency sideband and further cascades into a wave mode separated by the same frequency difference between the carrier wave and the lower sideband. We have also conducted a numerical experiment of random directional waves propagating through an ice field. Additional pressure terms due to flexural bending and mass-loading were added to the Higher Order Spectral Model, and the evolution of the directional spectra (JONSWAP and directional spread) was studied. While the mass-loading has practically no effect in a spectral downshift, with the bending effect, the energy seems to rapidly shift to both higher and lower frequencies due to three-wave interaction. We may conjecture that with a frequency-dependent attenuation, the lower energy transfer may survive and that partly explains the observed spectral evolution. We do not have a consistent explanation yet, but all these studies suggest that nonlinear evolution under sea ice may play a significant role in the evolution of waves under ice.

11:20-12:00 Emilian Părău (University of East Anglia)
The effect of vorticity and elasticity on the stability of travelling waves in deep water

We present some new results on the stability of nonlinear waves in deep water. Two cases are investigated in detail. First the effect of vorticity is analysed, using asymptotic and numerical methods based on conformal mappings to study the stability of the travelling waves. Then the stability of nonlinear hydroelastic waves to superharmonic and subharmonic perturbations is considered. This work is joint with Mark Blyth (UEA).

Lunch

14:00-14:40 Mats Ehrnström (Norwegian University of Science and Technology)
On two new constructions of solitary waves of the nonlinear and nonlocally dispersive Whitham equation

Solitary waves in dispersive and water wave equations are often constructed using either constrained minimisation or perturbative techniques around a trivial flow. In both cases, the resulting waves are typically small, because of nonlinear control. We present here two new proofs for existence of solitary waves in the nonlinear and nonlocal evolution equation

$$u_t + Lu_x + uu_x = 0, \qquad \mathcal{F}(Lu)(\xi) = \left(\frac{\tanh(\xi)}{\xi}\right)^{1/2} \mathcal{F}u(\xi),$$

also called the Whitham equation. The first proof is based on a priori estimates of periodic waves of all heights, and uses a limiting argument in the periodic to obtain a family of solitary waves up to the highest wave. The second uses a maximisation technique perhaps not earlier used in the water wave setting, where the dispersive part of the energy functional is maxmimised whereas remaining terms are held as a constraint in an Orlicz space constructed directly for this purpose. That is in many respects an L^p-based maximisation technique. We find in the second work small and intermediate-sized waves, although not necessarily a highest solitary wave.
The first work is joint with K. Nik and C. Walker; the second with A. Stefanov and M. N. Arnesen.

14:40-15:20 Tetsu Mizumachi (Hiroshima University)

Linear stability of elastic 2-line solitons for the KP-II equation

The KP-II equation was derived by Kadomtsev and Petviashvili to explain stability of line solitary waves of shallow water. Using Darboux transformations, we study linear stability of 2-line solitons whose line solitons interact elastically with each other. Time evolution of resonant continuous eigenfunctions is described by a damped wave equation in the transverse variable which is supposed to be a linear approximation of the local phase shifts of modulating line solitons.

Coffee break

15:40-16:20 David Ambrose (Drexel University)

The Hilbert transform and Birkhoff-Rott integral for non-decaying vortex sheets

The Birkhoff-Rott integral is a singular integral which expresses the velocity on a vortex sheet. The typical form of the Birkhoff-Rott integral converges for vortex sheets which are asymptotically flat at infinity, but in the spatially periodic case the integral may be carefully summed over periodic images. In the non-periodic, non-decaying case, it is less clear how to express this velocity. We will give a single new formula for the Birkhoff-Rott integral which converges in the decaying case, in the spatially periodic case, and in more general non-decaying, non-periodic cases. This has application to well-posedness theory for water waves with spatially quasiperiodic or uniformly local Sobolev data. In the well-posedness theory of water waves, one common tool is approximation of the Birkhoff-Rott integral by the Hilbert transform, and we will accordingly introduce a new representation of the Hilbert transform. This is a preliminary report of a work in progress.

16:20-17:00 Sarbarish Chakravarty (University of Colorado, Colorado Springs)

A class of rational solutions of KPI

A class of non-singular rational solutions of the Kadomtsev-Petviashvili (KP) I equation are explicitly constructed. The solutions have multiple peaks which form a two-dimensional wave pattern characterized by the partitions of a positive integer N, which is the number of peaks. The pattern formations are described by the roots of well-known polynomials arising in the study of rational solutions of Painlevé II and IV equations.

November 9 (Thu)

9:00-9:40 Theodoros Horikis (University of Ioannina)
Dark-bright soliton propagation under the perturbed Manakov system

We present a direct perturbation method for approximating the evolution of dark-bright solitons of Manakov equation under the influence of perturbations. Dark solitons -under the influence of perturbations- develop a shelf that propagates with speed determined by the background intensity, which, in our system, also affects the properties of the bright component. The dynamical properties of the solitons, and the shelf, are obtained from integral relations arising from the conservation laws of the Manakov equation, and exact analytical expressions are provided for the related fields accompanying the solitons during their propagation.

9:40-10:20 Mark Hoefer (University of Colorado, Boulder)
Modulation Theory and Shallow Water Wave Interaction Problems

Interactions of multidimensional nonlinear shallow water waves are ubiquitous in nearshore environments. In this talk, the KP-Whitham modulation equations will be used to investigate several wave interaction problems including i) soliton propagation past a corner (the Mach reflection and expansion problems) and ii) oblique soliton interaction with rarefaction/dispersive shock waves. The theoretical predictions are shown to agree quantitatively with numerical simulations of the KPII equation.

Coffee break

10:40-11:20 Yuji Kodama (Ohio State University)
On oblique interactions of KP solitons

We study oblique interactions of KP solitons using asymptotic perturbation theory. The initial wave consists of two half(or full)-line solitons. Then we identify exact KP solitons as asymptotic solutions of the initial value problems.

11:20-12:00 Nail Akhmediev (Australian National University)
Transformation of Plane waves into Periodic Waves

Mathematically, Bespalov–Talanov (BT) and Benjamin–Feir (BF) instabilities discovered more than 50 years ago (1966 and 1967, respectively) played a significant role in understanding nonlinear phenomena in optics and hydrodynamics. However, these are based on linearization of the nonlinear Schrödinger equation (NLSE). As such, the accuracy of their prediction is limited. In order to go beyond these limitations, we need to base our theory on exact solutions of the NLSE. Exact solutions in reality predict a wider frequency range for modulation instability than the BT and BF theories.

Group photograph

Lunch

Free discussion

Banquet

November 10 (Fri)

10:00-10:40 Sunao Murashige (Ibaraki University)
A numerical study on energy-conserving gravity currents past an air cavity

This work considers steady and two-dimensional motion of gravity currents past an air cavity which was originally investigated by Benjamin in 1968. First we derive some exact relations using some conservation laws, and compare them with internal bores and experimental results. Next we formulate the problem using some complex function theories and propose a representation of solution which is suitable for accurate computation. Some computed results are compared with Benjamin's approximate solution and solitary waves.
This is a joint work in progress with R. Camassa (Univ. North Carolina).

10:40-11:20 Ken-ichi Maruno (Waseda University)
Self-adaptive moving mesh schemes for nonlinear waves and numerical computations

Structure-preserving numerical methods of differential equations have been actively studied in the field of numerical analysis in recent years. In the field of integrable systems, solution structure preserving discretization of integrable systems, i.e. discrete integrable systems, have been actively studied after the monumental works of Hirota and Ablowitz-Ladik. We have proposed integrable self-adaptive moving mesh schemes of several soliton equations involving hodograph transformations such that the Camassa-Holm equation and the short pulse equation in which mesh intervals are automatically adjusted. I will talk about the construction of self-adaptive moving mesh schemes under general boundary conditions and its application to numerical computations. I also talk about integrable fully discrete self-adaptive mesh schemes and its application to numerical computations.

11:20-12:00 Yasuhiro Ohta (Kobe University)
On Airy function type solutions of KP equation

The KP equation has solutions expressed by the Airy function and its derivative. The fundamental solution of Airy function type represents the parabola wave front. The interactions of such waves are studied by using the Grammian expression of tau function. It is shown that the matrix elements of Grammian are simplified for special cases of parameters in the solutions.

Lunch

14:00-19:00 *Free discussion & Closing*