Workshop on Nonlinear Water Waves

In honor of Professor Mitsuhiro Tanaka on the occasion of his retirement

Date	:	May 23 (Wed) & 25 (Fri), 2018
Place	:	Research Institute for Mathematical Sciences, Kyoto University Room 111 of RIMS in North Campus
URL	:	http://murasige.sci.ibaraki.ac.jp/WS_Nonlinear_Water_Waves_RIMS_2018.html
Organizers	:	Takanori Hino (Yokohama National Univ.), Tatsuo Iguchi (Keio Univ.), Taro Kakinuma (Kagoshima Univ.), Takeshi Kataoka (Kobe Univ.), Ken-ichi Maruno (Waseda Univ.), Tetsu Mizumachi (Hiroshima Univ.), Sunao Murashige (Ibaraki Univ.), Yasuhiro Ohta (Kobe Univ.)

Program with abstract

May 23 (Wed)

13:00-13:10 Opening

13:10-13:40 Sunao Murashige (Ibaraki University)
Large-amplitude solitary waves on a linear shear current

This work considers large-amplitude motion of solitary waves propagating in permanent form on a linear shear current using the full Euler system and some long wave models. In particular, it is shown that exponentially decaying behavior of solutions in the outskirts of solitary waves dominate some characteristic properties such as the relation between the wave speed and the strength of shear current, the linear stability, and so on.

13:40-14:10 Yoshihiro Niwa (University of Tokyo)
Estimation of baroclinic tide energy available for deep ocean mixing based on three-dimensional global numerical simulations

Baroclinic tides are internal waves generated in the stratified ocean by tidal currents flowing over bottom topographies. They are considered to play an important role in ocean dynamics by providing energy for deep ocean turbulent mixing which drives the global thermohaline circulation.

In this study, the global distributions of semidiurnal and diurnal baroclinic tide energy are investigated using a three-dimensional hydrostatic sigma-coordinate numerical model. To examine the model sensitivity, a series of numerical simulations is conducted using various horizontal grid spacings of $1/20°$ to $1/5°$. For each simulation, the model topography is constructed by averaging the bathymetric data from ETOPO1 within each model grid cell.

The results of the simulation show that generation of energetic baroclinic tides are restricted to representative prominent topographic features. In particular, nearly half of the diurnal bacoclinic tide energy is excited along the western boundary of the North Pacific. The sensitivity experiments show that the conversion rate from tidal currents to baroclinic tides is very sensitive to the horizontal grid spacing as well as the resolution of the bottom topography. It is found that the conversion rate integrated over the global ocean is increased exponentially as the grid spacing is reduced. By extrapolating the calculated results, the global conversion rate in the limit of zero grid spacing is estimated to be about 1200GW (1GW=1012W). The amount of baroclinic tide energy dissipated in the deep ocean is estimated to be 600GW, which is consistent with the energy required for deep ocean mixing to sustain the thermohaline circulation estimated by the previous study of Webb and Suginohara (2001) (doi:10.1038/35051171).

Coffee break

14:30-15:20 Nail Akhmediev (Australian National University)

Classifying the rogue wave solutions of the nonlinear Schrödinger equation

Systematic classification of higher-order rogue wave solutions of the NLSE is given. These are nonlinear superpositions of the Peregrine solutions with multiple peaks of the wave field localized both in space and time. These solutions are structures with varying degrees of radial symmetry.

Coffee break

May 23 (Tue)

15:30-16:20 Wooyoung Choi (New Jersey Institute of Technology)

On spectral formulation for nonlinear water waves and their applications

The spectral formulation for nonlinear water waves introduced by Zakharov (1968) has been widely used to study broadbanded surface waves. In this talk, after addressing its relationship to the pseudo-spectral formulation of West et al. (1987) often adopted for numerical studies, some applications of the spectral formulation to uni-directional waves and resonant wave interactions will be discussed. In addition, the generalization of the spectral formulation to a two-layer system will be presented.

May 24 (Thu)

9:20-9:50 Taro Kakinuma (Kagoshima University)

A numerical calculation for internal waves over a slope or a mound

Internal waves, generated in a deep ocean, show deformation when they propagate over a continental shelf. In the present study, the set of nonlinear equations based on the variational principle, was solved numerically, to simulate the internal waves in two-layer systems over topography, with a slope or a mound, in consideration of both the strong nonlinearity, and the strong dispersion, of waves.

9:50-10:20 Takeshi Kataoka (Kobe University)

Transverse instability of surface solitary waves and breaking

The linear stability of finite-amplitude surface solitary waves with respect to transverse perturbations (three-dimensional perturbations) is investigated on the basis of the Euler set of equations. First, the linear stability to long-wavelength transverse perturbations is examined, and it is found that there exist transversely unstable surface solitary waves for the amplitude-to-depth ratio of over 0.713. This critical ratio is well below that (=0.781) for the longitudinal instability (instability to two-dimensional perturbations) obtained in the prior study. Next, the same transverse instability is examined numerically. Numerical results confirm the above analytical results in that the growth rates and the eigenfunctions of growing disturbance modes agree well with those obtained by the asymptotic analysis if the transverse wavenumbers of disturbances are small. The results for finite wavenumbers are also discussed. Finally, time evolution of transversely distorted solitary wave is simulated numerically in order to give clear intuitive picture of unstable wave motion.

May 24 (Thu)

Coffee break

10:30-11:20 Triantaphyllos Akylas (Massachusetts Institute of Technology)
Parametric subharmonic instability of internal waves: locally confined beams versus monochromatic wavetrains

Internal gravity wavetrains in continuously stratified fluids are generally unstable as a result of resonant triad interactions which, in the unviscid limit, amplify short-scale perturbations with frequency equal to one half of that of the underlying wave. This so-called parametric subharmonic instability (PSI) has been studied extensively for spatially and temporally monochromatic waves. We present an asymptotic analysis of PSI for time-harmonic plane waves with locally confined spatial profile, in an effort to understand how such wave beams differ, in regard to PSI, from monochromatic waves. For beams with general localized profile, it is found that triad interactions are not strong enough to bring about instability in the limited time that subharmonic perturbations overlap with the beam. On the other hand, for quasi-monochromatic wave beams whose profile comprises a sinusoidal carrier modulated by a locally confined envelope, PSI is possible if the beam is wide enough. An important exception arises when the beam frequency is nearly twice the inertial frequency due to background rotation; under this condition, PSI is possible for beams of general locally confined profile, as subharmonic perturbations of near-inertial frequency have small group velocity and stay in contact with the underlying beam longer, thus extracting more energy. The theoretical predictions are in keeping with numerical simulations and observations. (Joint work with Hussain Karimi and Boyu Fan.)

Coffee break

11:30-12:20 Roberto Camassa (University of North Carolina)
Hydrodynamic models and boundary confinement effects

Confinement effects by rigid boundaries in the dynamics of ideal fluids are considered from the perspective of long-wave models and their parent Euler systems, with the focus on the consequences of establishing contacts of material surfaces with the confining boundaries. When contact happens, it can be shown that the model evolution can lead to the dependent variables developing singularities in finite time. The conditions and the nature of these singularities are illustrated in several cases, progressing from a single layer homogeneous fluid with a constant pressure free surface and flat bottom, to the case of a two-fluid system contained between two horizontal rigid plates, and finally, through numerical simulations, to the full Euler stratified system. These illustrate the qualitative and quantitative predictions of the models within a set of examples chosen to illustrate the theoretical results.

Lunch

14:00-14:30 Takuji Waseda, Wataru Fujimoto and Yuki Kita (University of Tokyo)

Modulational instability in realistic directional seas

Research on freak/rogue waves in the ocean in the past decades has drawn attention of the physical oceanography, applied mathematics and engineering communities. Vast knowledge was gained on the role of the nonlinear processes in altering the Gaussian nature of the ocean waves. However, up to this point there is no consensus on whether the modulational instability plays a role in the formation of freak/rogue wave. In this talk, recent studies using the High Order Spectral simulation to understand the generation mechanism of the known freak wave events will be reviewed. The suggestion will be made that the values of the Kurtosis or the derived probability density function is not sufficient to identify the relative role of the nonlinear processes against the linear focusing effect. The life-time of the coherent wave group is a better indicator, and so is the shape of the freak wave (Fujimoto 2018 Ph.D. dissertation, Fujimoto et al. 2018, under review). We have also attempted to correlate meteorological events and the formation of a narrow directional spectrum that the nonlinear process can be more pronounced. An example of such study on explosive cyclone will be introduced (Kita et al. 2018, under review). In the talk, the key research accomplishments of Prof. Tanaka in utilizing the HOS model will be highlighted.

14:30-15:00 Naoto Yokoyama (Doshisha University / Kansai University) and Masanori Takaoka (Doshisha University)

Energy budget in stratified turbulence

In stratified turbulence, internal waves, eddies and vertically sheared horizontal flows coexist. Energy transfer among internal waves at the small wave-number range and that among eddies at the large wave-number range are well understood through the weak turbulence theory and Kolmogorov turbulence theory, respectively. It is still in debate whether or not the critical balance works well in the middle wave-number range where the transition between the wave-dominant region and the eddy-dominant region exists.

We have evaluated the energy budget in such region by using direct numerical simulations and will discuss generating and sustaining mechanisms of the coexistence in this talk.

Coffee break

15:10-16:00 Victor Shrira (Keele University)

Evolution of wind wave angular spectra: kinetic equations vs direct numerical simulations and observations

Wind waves in nature are inherently random. The challenge of describing evolution of random weakly nonlinear dispersive waves in fluids (and, in particular, water waves) remains a major open fundamental problem despite being intensively studied theoretically and experimentally for more than fifty years. In contrast to the classical hydrodynamic turbulence, for wave turbulence there is a well-established general formalism for treating weakly nonlinear wave fields that exploits smallness of nonlinearity and some other subtle assumptions. This approach leads to a closed equation for the second statistical momenta of the field – the *kinetic equation* (KE). The theory based upon the KE has been able to predict the major features of wave field evolution and is widely used; in particular, it underpins all practical wave modelling and forecasting. However, the basic question – to what extent the theory captures the actual behaviour of water waves — remains open. Here we address it by investigating discrepancies in the evolution of random weakly nonlinear water waves as described by the kinetic (Hasselmann) equation on the one hand, and, on the other, by direct numerical simulations based on the Zakharov equation for water waves and available experimental data. We examine wave field evolution both wind generated and without wind.The focus of the study is the long term evolution of the angular spectra. We trace evolution of second moments of the angular spectra. We report significant discrepancies between the predictions of the KE and both the DNS simulations field observations. The discrepancies suggest the presence and significance of coherent interactions not accounted for by the established closure for the kinetic equations. They also suggest a need to revisit the established practice of wave modelling and forecasting.

Coffee break

May 24 (Thu)

16:10-17:00 Mitsuhiro Tanaka (Gifu University)
Wave turbulence in a two-layer fluid system

The nonlinear energy transfer from the surface waves to interfacial waves in the wave turbulence of a two-layer fluid system is investigated numerically and theoretically. The wave-wave interaction which is mainly targeted in the study is the so-called "Class 3 resonance", i.e., a triad resonance which is possible among two surface waves and one interfacial wave all propagating in the same direction. The direct numerical simulation (DNS) shows that the spectrum evolves with a timescale which is consistent with the traditional wave turbulence theory (WTT) in most part of the wavenumber. The DNS also shows however that a sharp peak appears spontaneously in the energy spectrum of the surface waves around the wavenumber k_c corresponding to the minimum of the wavenumbers for which the Class 3 resonance is possible, and that the peak grows in a timescale much faster than that predicted by WTT. This result of DNS is discussed in relation to the breakdown at k_c of the kinetic equation given by WTT.

Banquet (invitation only)

May 25 (Fri)

9:20-9:50 Ken-ichi Maruno (Waseda University)
The interactions of dark line solitons in the Davey-Stewartson II system

The Davey-Stewartson (DS) system is a (2+1)-dimensional integrable system which describes weekly nonlinear water waves. It is known that the DS II system has N-dark line soliton solutions and resonance solutions which show complicated interaction patterns called soliton reconnection phenomena. However, the detail of interactions of dark line solitons and resonance solutions has not been studied yet. In this talk, I will discuss the detail of interactions of dark line solitons in the DS II system. This is joint work with Daichi Suyama and Arata Nagahara.

9:50-10:20 Yasuhiro Ohta (Kobe University)
Time-localized solutions for some soliton equations

Some soliton equations admit solutions localized in time, such as Akhmediev breathers and rational rogue wave solutions. These types of time-localized solutions are constructed explicitly for some soliton equations by using direct method in soliton theory. Algebraic structure of the solutions is also discussed.

May 25 (Fri)

Coffee break

10:30-11:20 Yuji Kodama (Ohio State University)
Mach Reflection of a Solitary Wave: Revisited
Part I: Theory

We will start to derive the KP equation including the next order corrections from the three-dimensional Euler equation for shallow water waves. Then we develop the normal form theory for the higher order KP equation and show that the higher order corrections can provide an excellent remedy for the strict assumptions, such as the quasi-two dimensionality, for the KP equation. The talk will also give a brief introduction to the KP solitons and their classification for the application to the Mach reflection phenomena.

Coffee break

11:30-12:20 Harry Yeh (Oregon State University)
Mach Reflection of a Solitary Wave: Revisited
Part II: Experiments

The laboratory and numerical experiments are presented for Mach reflection of an obliquely incident solitary wave at a vertical wall. The numerical model is based on the pseudo-spectral method for the full Euler formulation: the model is an extension of that used by Tanaka. With the aid of a laser sheet in the laboratory, the wave profiles are measured optically in sub-millimeter precision. Discrepancies reported in previous works are now substantially improved, partly because of the higher-order KP theory (Part I) and in part the advancement in computational power and laboratory instrumentations.

Group Photo

Lunch

14:30-15:20 David Lannes (The Université de Bordeaux)
The Boussinesq equations with a floating obstacle

The aim of this talk is to propose and analyse a model for the description of dispersive surface water waves in shallow water, in the presence of a floating structure. More precisely, we describe the evolution of the waves using a Boussinesq type model in the exterior domain (outside the floating object). Underneath the object, the equations are of a different nature since the elevation of the surface of the water is now constrained, and the pression exerted by the fluid on the object is unknown. Energetical consideration are used to determine it. We then show that this problem can be reduced to a transmission problem for the Boussinesq system, with a nonstandard transmission condition. We then prove the well posedness of this transmission problem, the main difficulty being the appearance of a new phenemonen with respect to the hyperbolic (non dispersive) case, namely, the presence of dispersive boundary layers. This is a joint work with D. Bresch and G. Métivier.

Coffee break

15:30-16:00 Yoshimasa Matsuno (Yamaguchi University)

The N-soliton formulas for a multi-component modified nonlinear Schrödinger system with nonzero boundary conditions

We consider the integrable multi-component modified nonlinear Schrödinger system

$$\mathrm{i}\, q_{j,t} + q_{j,xx} + \mu \left(\textstyle\sum_{s=1}^{n} \sigma_s |q_s|^2\right) q_j + \mathrm{i}\gamma \left[\left(\textstyle\sum_{s=1}^{n} \sigma_s |q_s|^2\right) q_j\right]_x = 0, \quad (j = 1, 2, ..., n),$$

where $\sigma_s = \pm 1$, $q_j = q_j(x,t) \in \mathbb{C}$ and $\mu, \gamma \in \mathbb{R}$. In particular, we construct the N-soliton solutions for the system under the following two types of boundary conditions:

1) Plane-wave boundary conditions.

$$q_j \sim \rho_j \exp \mathrm{i} \left(k_j x - \omega_j t + \phi_j^{(\pm)}\right), \quad x \to \pm\infty, \quad \rho_j,\ \phi_j^{(\pm)} \in \mathbb{R}, \quad (j = 1, 2, ..., n),$$

$$\omega_j = k_j^2 - \mu \sum_{s=1}^{n} \sigma_s \rho_s^2 + \gamma \left(\sum_{s=1}^{n} \sigma_s \rho_s^2\right) k_j, \quad (j = 1, 2, ..., n).$$

2) Zero and plane-wave boundary conditions.

$$q_j \sim 0, \quad x \to \pm\infty, \quad (j = 1, 2, ..., m),$$

$$q_{m+j} \sim \rho_j \exp \mathrm{i} \left(k_j x - \omega_j t + \phi_j^{(\pm)}\right), \quad x \to \pm\infty, \quad (j = 1, 2, ..., n-m),$$

$$\omega_j = k_j^2 - \mu \sum_{s=1}^{n-m} \sigma_s \rho_s^2 + \gamma \left(\sum_{s=1}^{n-m} \sigma_s \rho_s^2\right) k_j, \quad (j = 1, 2, ..., n-m).$$

A direct method is developed to transform the above system into the system of bilinear equations through appropriate dependent variable transformations. The construction of the N-soliton solutions of the bilinear equations is performed by means of an elementary theory of determinants.

May 25 (Fri)

16:00-16:30 Tetsu Mizumachi (Hiroshima University)

On the phase shift of line solitary waves for the KP-II equation

Since when the KP equations are derived by Kadomtsev and Petviashvili in 1970, line solitons for the KP-II equation are considered to be stable. Recently, 1-line solitons turned out to be stable in a relatively weak sense. In this talk, I will report that the sup norm of the phase shift remains although its growth rate in $L^2(\mathbb{R}_y)$ was known to be equal to or greater than $t^{1/4}$.

16:30-17:00 Tatsuo Iguchi (Keio University)

Kakinuma model for internal gravity waves in the rigid-lid case

We consider the motion of internal gravity waves at the interface between two immiscible inviscid fluids of different densities in the case where the top water surface of the upper layer is assumed to be flat. As in the case of gravity water waves, the basic equations have a variational structure with a Lagrangian in terms of the surface elevation of the interface and the velocity potentials of the two fluids. Kakinuma model is the Euler-Lagrange equation for an approximate Lagrangian, which is derived from the original Lagrangian by approximating the velocity potentials appropriately. We show basic structures of the Kakinuma model, especially, the linear dispersion relation, which implies that the Kakinuma model would be a good approximation to the original model in the shallow water regime. Although the initial value problem to the original model is ill-posed, the problem to the Kakinuma model turns out to be well-posed in an appropriate condition on the initial data.

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