Kobe Workshop on Nonlinear Water Waves

https://murasige.sci.ibaraki.ac.jp/WS_Nonlinear_Water_Waves_Kobe_2025.html

Date : July 7 (Mon) – 10 (Thu), 2025

Place : Hotel Kitano Plaza Rokko-so (Kobe, Japan)

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Program with title & abstract

July 7 (Mon)

10:30-10:40 *Opening*

10:40-11:20 Takeshi Kataoka (Kobe University)

Surface waves due to subcritical flow past topography: an exponential-asymptotics solution

The downstream radiation of steady surface gravity waves by a subcritical flow over locally confined, smooth, finite-amplitude, long topography is analyzed based on potential flow theory. Due to the mismatch in lengthscale between the topography and the wavelength of the radiating waves, the wave amplitude becomes exponentially small. This necessitates the use of exponential (beyond-all-orders) asymptotics. The obtained solution shows that, as the topography amplitude increases, the wave amplitude grows very rapidly. These results are confirmed by numerical solutions of the full steady water wave equations, which demonstrates that the computed wave resembles a steep Stokes wave, despite the wavelength being much shorter than the topography.
This is joint work with T. R. Akylas.

11:20-12:00 Amin Chabchoub (Okinawa Institute of Science and Technology)
Hydrodynamic Manakov solitons and breathers

The hydrodynamic and integrable Manakov equation can rise from the coupled nonlinear Schrödinger equation, when the crossing angle of two wave fields correspond to particular values. We report observations of exact Manakov envelope solitons and breathers in a water wave basin. Considering the challenges in building the appropriate experimental setup to observe the clean evolution of crossing wave envelopes, the data show an excellent agreement with the theory. The experiments shed a new light on rogue wave formation in the ocean, particularly, when two wave systems are at play.

Lunch

14:00-14:40 Ken-ichi Maruno (Waseda University)
An integrable semi-discretization of the 2-component Hunter-Saxton equation

The Hunter–Saxton (HS) equation is an integrable partial differential equation that describes the propagation of weakly nonlinear orientation waves in the director field of a massive nematic liquid crystal. This talk will begin with an overview of its connection to the Camassa–Holm (CH) equation, followed by a discussion of their two-component generalizations — the two-component HS and two-component CH equations. I will then present our main result: a novel integrable semi-discretization of the two-component HS equation.
This is joint work with Ayako Hori, Yuta Tanaka, and Yasuhiro Ohta.

14:40-15:20 Justin Cole (University of Colorado, Colorado Springs)
Dispersive Spiral Waves

Spiral waves are phenomena frequently studied in reaction-diffusion systems. While less frequently examined, spiral waves are also observable in dispersive systems. Originally motivated by optical metamaterials, spiral waves are studied in a variety of classic (2+1)D partial differential equations, such as the Dirac and Klein-Gordon equations. Moreover, the water wave equations also support spiral wave solutions. Spiral wave dynamics and their long time asymptotics are examined.

Coffee break

15:40-16:20 Dag Nilsson (Mid Sweden University)
Axisymmetric capillary solitary waves with vorticity and swirl

We consider steady, axisymmetric solitary waves propagating under the influence of surface tension, allowing for general vorticity and swirl. The governing equations can be formulated as an elliptic free boundary value problem. We reformulate these equations as a dynamical system in which an unbounded spatial coordinate plays the role of time (a spatial dynamics approach). By using the centre manifold theorem, we prove the existence of solitary waves for special choices of vorticity and swirl. Additionally, we investigate the existence of generalized solitary waves.
This talk is based on a work in progress with Stefano Böhmer (Lund University) and Dan Hill (Saarland University).

16:20-17:00 Samuel Walsh (University of Missouri)
Extreme internal waves

In this talk, we will discuss some recent work in progress on two types of extreme internal waves: overturning internal waves and gravity currents. These are front-type traveling wave solutions to the two-layer free boundary Euler equations in two dimensions. The velocity field in each layer is assumed to be incompressible and irrotational, and it limits to distinct laminar flows upstream and downstream. A constant gravitational force acts on the waves, but surface tension is neglected. In an early work, we proved that there exists two large-amplitude families of solutions: a curve of depression bores and a curve of elevation bores. Both bifurcate from the trivial solution where the interface is flat.
We can now show that along the depression bore family, overturning must occur in that the interface separating the layers develops a vertical tangent. This type of behavior was first observed over 40 years ago in computations of internal gravity waves and gravity water waves with vorticity via numerical continuation, but a rigorous proof has been stubbornly elusive. Time permitting, we will also discuss the situation along the elevation bore family, which are expected to limit to a gravity current wherein the internal interface meets with upper wall.
This is joint work with Robin Ming Chen and Miles H. Wheeler.

17:00-17:40 Mark Groves (Universität des Saarlandes)

Solitary wave solutions to the full dispersion Kadomtsev-Petviashvili equation

The KP-I equation

$$u_t + m(D)u_x - 2uu_x = 0 \quad ,$$

where $m(D)$ is the Fourier multiplier operator with multiplier

$$m(k) = 1 + \frac{{k_2}^2}{2{k_1}^2} + \frac{1}{2}(\beta - \frac{1}{3}){k_1}^2 \quad , \qquad (\star)$$

arises as a weakly nonlinear model equation for gravity-capillary waves with strong surface tension (Bond number $\beta > 1/3$). It has recently been shown by Liu, Wei and Yang that $(\star)$ has an infinite family $\{\zeta_k\}$ of symmetric 'lump' solutions of the form

$$\zeta_k(x, y) = -2\partial_x \log \tau_k(x, y) \quad , \qquad k = 1, 2, ...,$$

where τ_k is a polynomial of degree $k(k+1)$ given by an explicit formula. They also show that ζ_1 and ζ_2 are nongenerate and announce that the same is true for ζ_k, $k \geq 3$ (details to be published in a later paper).

Recently there has been interest in the full dispersion KP-I equation

$$u_t + \tilde{m}(D)u_x - 2uu_x = 0 \quad ,$$

obtained by retaining the exact dispersion relation from the water-wave problem, that is, replacing m by

$$\tilde{m}(k) = \left((1 + \beta|k|^2) \frac{\tanh|k|}{|k|} \right)^{1/2} \left(1 + \frac{{k_2}^2}{{k_1}^2} \right) \quad .$$

In this talk, I show that the FDKP-I equation also has a family of symmetric fullly localised solitary waves which are obtained by casting it as a perturbation of the KP-I equation and applying a suitable variant of the implicit-function theorem.

This project is joint work with Mats Ehrnström (NTNU, Norway).

July 8 (Tue)

9:00-9:40 Sunao Murashige (Ibaraki University)

Gravity currents and steep internal fronts

This work considers an irrotational plane motion of a gravity current along the horizontal top wall of a channel as the limit of internal fronts generated at the interface of the two-layer flow with increase of amplitude. First, numerical examples for an initial-value problem of partial-depth lock-exchange flow demonstrates that the long wave approximation is not valid for steep internal fronts which approach the gravity current. Next, using the steady-state approach, it is shown that a class of steady internal fronts with a specific upstream condition approaches the steady gravity current with decrease of the upstream depth ratio. In addition, we propose an analytical model based on Benjamin's approximate solution which produces the limiting behaviour of steep internal fronts.
This is joint work with Roberto Camassa.

9:40-10:20 Mark Blyth (University of East Anglia)

Stability of waves on fluid of infinite depth with constant vorticity

We examine the stability of two-dimensional waves on the surface of inviscid fluid of infinite depth in the presence of a constant vorticity shear current. The basic state solution corresponds to a periodic travelling wave that is moving at constant speed. The branch of periodic travelling waves is remarkable in that it can be described by an exact solution (Hur & Wheeler 2020). Our concern is with the linear stability of these travelling waves. We study this first for small amplitude waves and we identify resonant triad interactions leading to instability for non-zero wave amplitude. Stability for arbitrary amplitude waves is determined numerically using a Floquet-Fourier-Hill approach and a collocation method. The effect of weak gravity is also considered.

Coffee break

10:40-11:20 Mitsuhiro Tanaka (Gifu University)
Numerical study on the fast spectral evolution in a simplified two-layer fluid system

It is known that for a two-layer fluid system, the kinetic equation governing the evolution of the spectrum of the wave field given by the Hasselmann-Zakharov-type standard wave turbulence theory breaks down due to the existence of a "double resonance", and the spectrum can evolve on a time scale much faster than that predicted by the standard wave turbulence theory. In this study, using a simplified model for a two-layer fluid system, we numerically examine the applicability of the generalized kinetic equation for such a situation.

11:20-12:00 André Nachbin (Worcester Polytechnic Institute)
Water-wave propagation on graphs

We present a weakly nonlinear, weakly dispersive Boussinesq system for water waves propagating on a 1D branching channel, namely for studying reflection-transmission on a metric graph. Our graph model uses a new nonlinear compatibility condition at the vertex which improves reflection-transmission properties, and therefore generalizes the well-known Neumann-Kirchhoff condition. The model includes forking-angles in a systematic fashion. Our vertex condition is formulated by looking also at solutions of the 2D (parent) fattened graph model, namely a graph-like domain with a small lateral width. We present numerical simulations comparing solitary-wave-propagation on the 1D (reduced) graph model with the respective results of the 2D model, where a compatibility condition is not needed at the forked region. We will comment on ongoing studies, in particular regarding the importance of including angle-information, a feature not present in most waves-on-graph models.

Lunch

14:00-14:40 Magda Carr (Newcastle University)

Experimental & numerical modelling of internal solitary waves

Internal solitary waves (ISWs) are nonlinear, finite-amplitude waves that propagate along density interfaces in stratified fluids, driven by a balance between nonlinear steepening and linear dispersion. Ubiquitous in coastal oceans, straits, and fjords, ISWs play a critical role in oceanic mixing, sediment resuspension, and benthic boundary layer dynamics, with implications for both environmental processes and offshore engineering.

This presentation explores both experimental and numerical approaches to modelling ISWs. Laboratory experiments are conducted in a stratified wave flume, with flow visualization achieved through particle image velocimetry (PIV). Complementary numerical simulations solve the incompressible Navier-Stokes equations using a spectral parallel solver to capture wave dynamics and breaking behaviour.

A novel diagnostic tool will be presented to analyse mixing in such stratified fluids. By employing paired histograms of user-defined variables, the method identifies and visualizes regions of active mixing, revealing differences in mixing extent and advection across various ISW breaking regimes.

The talk will focus on recent research highlights including ISW shoaling over sloping topography and interactions with floating bodies. If time a summary of past work and current interests will also be presented on areas including mode-2 ISWs.

14:40-15:20 Martin Erinin (University of Michigan)

The effect of surfactants on plunging breakers: surface profile evolution and droplet generation

Experimental studies are conducted to study the free surface profile evolution and droplet generation of a deep-water plunging breaker in the presence of three bulk concentrations of the soluble surfactant Triton X-100 (zero, below the critical micelle concentration (CMC) and above the CMC). The breakers are generated by a programmable wave maker that is set with a single motion profile that produces a highly repeatable dispersively focused quasi-2D wave packet with a central frequency of $f_0 = 1.15$ Hz and a corresponding wavelength of $\lambda_0 = 1.18$ m (by linear theory). The wave profiles are measured with a cinematic LIF technique and droplets are measured using an inline holographic technique, both operating at 650 Hz. It was found that a plunging breaker is formed at all surfactant conditions. However, at the intermediate surfactant condition, the jet becomes convoluted before impact and the water surface below the jet appears to fall on the forward wave face before impact. The positions, diameters ($d \geq 100$ μm), times and velocities of droplets generated in the three surfactant cases are measured as the droplets move up across a prescribed horizontal measurement plane. It is found that the surfactants have strong effects on the droplet production mechanisms and the droplet number, diameter, and velocity distributions. One example is the droplets produced by the closure of the crater generated between the upper surface of the plunging jet and the splash that it produces. This crater closes rapidly in the TAP and TX6 cases but not in the TX1 case, resulting in much fewer droplets produced in TX1 than in TAP and TX6.

Co-authors : Chang Liu (University of Maryland), Wouter Mostert (University of Oxford), Xinan Liu (University of Maryland), Luc Deike (Princeton University), James Duncan (University of Maryland)

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Coffee break

15:40-16:20 Yohei Onuki (Kyushu University)

Breaking of internal waves simulated in a distorted domain model

Internal gravity waves play a crucial role in transporting energy and driving mixing in the ocean interior. Their breaking, governed by nonlinear interactions, leads to turbulence and irreversible mixing. To investigate these processes, we develop a numerical approach that isolates a small domain influenced by large-scale flows through periodic distortions, allowing us to focus on wave breaking and the resulting turbulence while implicitly accounting for energy input from unresolved outer fields.

We apply this approach to two canonical nonlinear wave problems. First, the parametric subharmonic instability of a monochromatic internal wave is simulated in a tilted, oscillating frame. This leads to the growth of subharmonic modes, followed by secondary instabilities and small-scale turbulence, with potential energy efficiently transferred from the primary wave to mixing. Second, we investigate elliptic instability by rotating the model domain along elliptic streamlines. Depending on the instability growth rate, we observe either rapid, direct wave overturning and strong turbulence, or gradual energy redistribution through weak interactions, eventually forming a Garrett–Munk-like spectrum with finer-scale breaking.

Our results highlight how nonlinear instabilities drive internal wave evolution and turbulence, providing new insights relevant to geophysical and fluid dynamics systems.

References and collaborators:

- Onuki, Y., S. Joubaud, and T. Dauxois. "Simulating turbulent mixing caused by local instability of internal gravity waves." Journal of Fluid Mechanics 915 (2021): A77.
- Onuki, Y., S. Joubaud, and T. Dauxois. "Breaking of internal waves parametrically excited by ageostrophic anticyclonic instability." Journal of Physical Oceanography 53.6 (2023): 1591-1613.

16:20-17:00 Hidetaka Houtani (University of Tokyo)

Experiment on nonlinear design irregular waves for predicting extreme values of ship response in waves

In this study, nonlinear design irregular waves (DIWs) evolving in time were generated in a wave basin. DIW is also known as the response-conditioned wave. These waves express the most likely wave event for a given extreme value of a certain response of a ship or offshore structure in waves. The objective of this study is to experimentally demonstrate that DIWs can be accurately reproduced in a wave basin by accounting for the nonlinear wave evolution appropriately. To this end, this study employs the "HOSM+FORM" approach, which combines the higher-order spectral method (HOSM) and the first-order reliability method (FORM) to obtain DIWs. The HOSM is used to simulate the nonlinear evolution of waves, while the FORM is used as a detector for a DIW associated with a given extreme ship response value. This study targets the vertical bending moment of a container ship. An experiment was conducted in a wave basin to generate the DIWs obtained by the HOSM+FORM. The experimental results demonstrated that the DIWs can be accurately reproduced in the wave basin without the need to correct the wave maker signals, thanks to the incorporation of the nonlinear wave evolution effect in the DIW estimation and wave maker signal calculation phases.

18:30–20:30 *Banquet* at "Sakabayashi"

July 9 (Wed)

10:00-10:40 Taro Kakinuma (Kagoshima University)

Tsunami-height reduction using a very large floating structure

Tsunami-height reduction using a very large floating structure (VLFS) is simulated, considering the interaction between water and a floating thin plate, with a numerical model based on both the variational principle for water waves and the classical theory for the oscillation of an elastic plate.

10:40-11:20 Zhan Wang (Chinese Academy of Sciences)

Dynamics of ferrofluid jets: the Hamiltonian framework

Ferrohydrodynamics deals with the mechanics of fluid motion influenced by strong forces of magnetic polarization. Developing an understanding of the consequences of these forces enjoys wide usage in industries, including magnetic resonance imaging, dynamic loudspeakers, magneto-optical sensors, and heat transfer or dissipation. This talk focuses on the stability and dynamics of solitary waves propagating along the surface of an inviscid ferrofluid jet from analytical and numerical aspects. First, we give a detailed proof of the Hamilton principle for the axisymmetric system, as well as the canonical variables. Next, the homogeneous expansion of the Dirichlet-Neumann operator (DNO), slightly differing from Guyenne & Parau (2016), is obtained. Finally, a systematic procedure is proposed to derive model equations of multiple scales in various possible limits from the full potential problem in the Hamiltonian/Lagrangian framework. In particular, based on the Hamiltonian perturbation theory, we propose a simplified model with full dispersion by truncating the DNO expansion at the cubic order for the kinetic energy. It is shown that the model agrees well with the full Euler equations for the speed-amplitude and speed-energy bifurcation curves and wave profiles. Based on the model, we examine analytically the stability properties of axisymmetric solitary waves subject to longitudinal disturbances. Our analytical result, consistent with that obtained by Saffman (1985), indicates that in the axisymmetric system, the stability exchange for solitary waves (namely, the superharmonic instability) also occurs at the stationary points of the speed-energy bifurcation curve.

11:20-12:00 Roberto Camassa (University of North Carolina)

Hydrodynamic models and boundary confinement effects

Confinement effects by rigid boundaries in the dynamics of ideal fluids are considered from the perspective of long-wave models and their parent Euler systems, with the focus on the consequences of establishing contacts of material surfaces with the confining boundaries. When contact happens, it can be shown that the model evolution can lead to the dependent variables developing singularities in finite time. The conditions and the nature of these singularities are illustrated in several cases, progressing from a single layer homogeneous fluid with a constant pressure free surface and flat bottom, to the case of a two-fluid system contained between two horizontal rigid plates, and finally, through numerical simulations, to the full Euler stratified system. These illustrate the qualitative and quantitative predictions of the models within a set of examples chosen to illustrate the theoretical results.

Group photo & Lunch

14:00-14:40 Yeunwoo Cho
(Korea Advanced Institute of Science and Technology (KAIST))
Breather solutions of a quadratic nonlinear beam equation

We consider a quadratic nonlinear Bernoulli-Euler beam equation supported by a distributed-spring elastic (Winkler) foundation, which is a simple model for flexural waves on an ice sheet floating on water. The linear dispersion relation of the quadratic nonlinear beam equation features a minimum at a nonzero wavenumber. Below this minimum, there exist two kinds of solitary wave solutions; elevation and depression solitary waves. By assuming slowly varying envelope solutions in the small-amplitude weakly nonlinear limit, it is found that elevation solitary waves are unstable and depression solitary waves are stable. In numerical simulations, however, for given initial perturbations, depression solitary waves turn out to be unstable in that they show periodic breathing behaviors, i.e., breather solutions. Furthermore, it is found that the period of a breather solution decreases as the magnitude of the initial perturbation increases.
This work was supported by National Research Foundation of Korea (NRF-2023R1A2C1003600).

14:40-15:20 Paul Milewski (The Pennsylvania State University)
Embedded Solitary Waves in the Euler equations: three-layer stratified flows

The ocean and atmosphere are density stratified fluids. A wide variety of gravity waves propagate in their interior, redistributing energy and mixing the fluid, affecting global climate balances. Stratified fluids with narrow regions of rapid density variation with respect to depth (pycnoclines) are often modelled as layered flows. We shall adopt this model and examine horizontally propagating internal waves within a three-layer fluid, with a focus on mode-2 waves which have oscillatory vertical structure and whose observations and modelling have only recently started. Mode-2 waves (typically) occur within the linear spectrum of mode-1 waves (i.e. they travel at lower speeds than mode-1 waves), and thus mode-2 solitary waves are generically associated with an unphysical, resonant, mode-1 infinite oscillatory tail. We will show that these tail oscillations can be found to have zero amplitude, thus resulting in families of localised solutions (so called embedded solitary waves) in the Euler equations. This is the first example we know of embedded solitary waves in the Euler equations. Their existence implies that mode-2 waves are probably longer lived in nature that previously thought.

Coffee break

15:40-16:20 Philippe Guyenne (University of Delaware)
Modeling and simulation of wave propagation and attenuation in sea ice

This talk will give an overview of my recent research on wave-ice interactions. Mathematical models are developed to describe wave propagation and attenuation in sea ice. Various mechanisms are taken into account to represent the complex nature of sea ice and its effects on wave dynamics. These predictions range from linear to weakly nonlinear models. Comparison with other existing theories and experimental data will be presented. Direct numerical simulations of nonlinear waves will also be shown, highlighting various configurations with a continuous or fragmented ice cover.

16:20-17:00 Wooyoung Choi (New Jersey Institute of Technology)
High-order strongly nonlinear long wave approximation

Nonlinear long waves in shallow water have been often described by weakly nonlinear models. Considering that the number of extreme events increases, strongly nonlinear models have recently attracted much attention. In this talk, a high-order strongly nonlinear long wave model will be introduced and its well-posedness and solitary wave solution will be discussed along with its generalization to shallow water with variable bottom and density stratification.

July 10 (Thu)

9:00-12:00 *Free discussion*

Lunch

14:00-19:00 *Free discussion & Closing*